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Evaluation of Failure Criteria in Branch Members Under Torsion and Bending Moment

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Abstract. Tree assessment relies on identifying and assessing structural conditions to predict failure. The tree is a complex structure made of live composite materials where multi-axial stress conditions exist in the interaction of the branches between external and internal forces. Failure criteria included bending moment and torsion, while also considering the wood as an orthotropic material. Ash (*Fraxinus* spp.) tree branches that failed in natural conditions demonstrated to be subject to both torsion and bending moment upon failure. Maximum stress values measured in controlled failure experiments demonstrated excellent agreement with the criteria used in the study, thus obtaining a curve extending from pure bending moment failure, pure torsion failure, and combination of both. From these experiments, the ultimate values determined for the tree species tested were $\overline{X}$ = 75.5 MPa for pure bending and $\overline{Z}$ = 11.2 MPa for pure torsion, which agrees well to previous reports. The methodology shown in this study may be applied to any tree species.

Key Words. Ash; Bending Moment; Failure; Failure Criteria; *Fraxinus uhdei*; Torsion; Orthotropic Material.

The strength of branches has been studied in the past, in most of the previous investigations, this has occurred in the area of the strength of tree branches. For example, a breaking load applied in a bending moment to the branch (Miller 1959; Smiley et al. 2000; Kane 2007). There are studies where breaking loads, due to bending and torsion loads, are applied to the branch separately (Niklas and Spatz 2010). In all studies, there is there is no information of the branch failure for the combined loading of bending and torsion simultaneously. Therefore, the aim of this research is to determine, experimentally, the combination of loading that will lead to branch failure. This study was performed on branches that were structural sound.

To measure integrity of wood, several instruments have been used over the years, including fractometers, ultrasound equipment, and microdrilling equipment. Fractometers evaluate wood decay (Mattheck et al. 1994; Mattheck et al. 1995), allowing an assessment of tree integrity. Until now, it is the only instrument that provides the fracture moment and the angle of failure for the sample and provides accurate measurements of the integrity of the wood, suggesting a reference to determine the failure potential (Mattheck et al. 1995; Matheny et al. 1999); this method is considered invasive.

Resistance microdrilling measures the relative resistance (i.e., drilling torque) of a material, as a rotating needle is driven into the wood at a constant speed. Changes in wood resistance are displayed on a graph as changes in amplitude. Areas of prolonged low resistance indicate decay, cavities, or cracks. Because it requires drilling into the tree, this test is considered minimally invasive.

Ultrasounds or sound waves (see McCracken 1985; Mattheck and Bethge 1993; Yamamoto et al. 1998; Wang et al. 2004; Wang and Allison 2008) are used to determine internal structural defects hidden from view within the tree and show an image of the geometry of the cross section, the latter being an important parameter to quantify the stress values that external forces will induce upon the tree.

None of the current measurement tools available provide information about the effect of the combination of both torsion and bending moment applied simultaneously. There are studies of mechanical failure in the branches that consider bending moment (Ennos et al. 2009; Van Casteren et al. 2011), but the effect of the combination of the torsion and bending moment loads on the branch failure is not considered.

Research that considers both bending and torsion in branch failure is a more realistic approach, as both are present in the tree in natural conditions. The branch's own weight, inertial forces, wind interaction with leaves and branches, and possible forces due to an external element like a rope attached to a branch, or an external boundary to the branch, like that of a building or wall, represent different scenarios where both torsion and bending might be present. To avoid unpredictable branch failure, it is important to identify if torsion is present in the tree branches, determine its effect on the structural integrity of the branch, and finally, identify which values characterize the ultimate stress value the branch can withstand to forces.

The current investigation originated after visually comparing branch cross sections that failed naturally against those that failed in a controlled environment. Three cases were reproduced: pure bending, pure torsion, and combined. The section of the branches that failed in real conditions did not match with the characteristics of what pure bending moment generates in the failure zone.

In this paper, researchers present a study of failure in branches by calculating the internal stresses due to external forces of bending moment and torsion at the point of failure; this is achieved by probing a destructive test to the branch under both loads.

Furthermore, this study aims to evaluate two different failure criteria, and to explain the distinction between the applied forces classified as axial force, bending moment, and torsion.

MATERIALS AND METHODS

Of the variables to consider in most structural failure criteria, the stress tensor contains up to six independent stress values. If the structural element is considered an axial element, like a beam, which is a reasonable assumption for a branch, then researchers can consider three types of forces than can be applied, namely axial, bending, and torsion. Axial forces consist in pulling or compressing the branch, where the force direction is parallel to the branch core as shown in Figure 1a. Bending moment is present from the branch's own weight as well as when there is an external weight on the branch (e.g., snow) and whose direction is perpendicular to the branch cross section, as depicted in Figure 1b. Lastly, torsion, being a consequence of torque, is directed around the branch axis direction as seen on Figure 1c, where the torsion symbol is expressed as a line along the rotation axis.

The primary forces that break a branch consist of two element forces. One force is perpendicular to the branch, generating bending moment, the second is torque acting on the axis of the branch, creating torsion by the subsequent smaller branches that generate a force at distance by being attached to the main branch, as seen in Figure 2.

Shear stresses appear in a cantilever beam when a force perpendicular to the beam axis is applied (Figure 1b); this perpendicular force is commonly known in strength of materials as shear force, which generates a shear stress at the neutral fiber of the branch (the neutral fiber is where there are no longitudinal strains, for a symmetric section it occurs at the geometric centroid). However, this shear force does not induce shear stress at the cross section of the branch where the maximum stress is located, due to the fact that bending occurs, thus, the shear stress does not need to be considered (Ferdinand et al. 2011)

The stress tensor for the critical point when the branch is subjected to a bending moment and torsion is shown in Figure 3 and is labeled as Point 1. At said point, the maximum stress

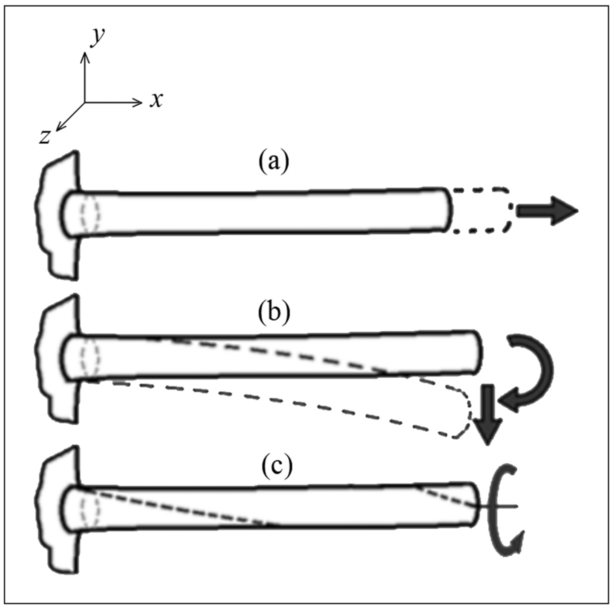

Figure 1. (a) Axial load acting along the axis of the branch, b) bending moment tends to rotate the branch around the *z* axis, and c) torsion tends to rotate the branch around the *x* axis.

value is reached when bending moment is applied and is in tension, as opposed to the bottom point, which is in compression. Furthermore, shear stress is added to the critical point of the branch because of the principle of superposition.

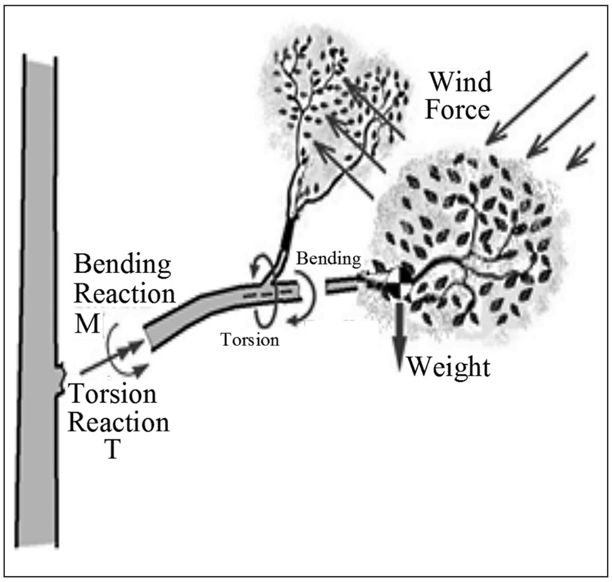

Figure 2. Schematic representation of the induced forces in a branch. Moment is present in the branches due to weight and wind flowing through leaves, torsion is present in the branch from adjacent branches and is also subject to weight and wind forces.

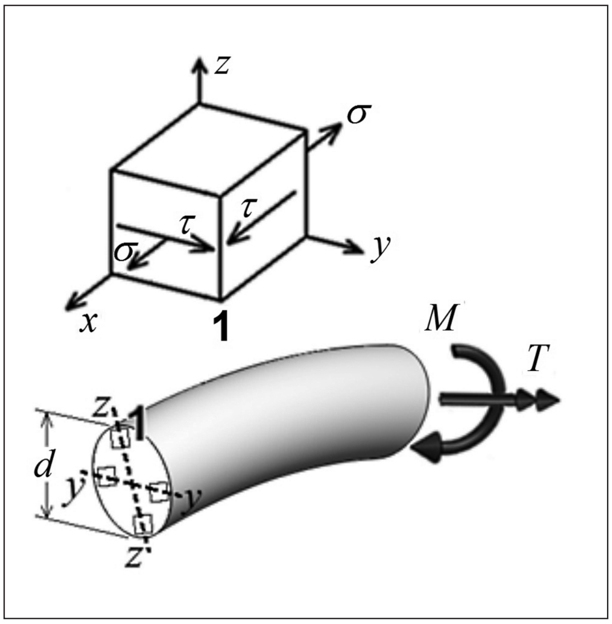

Figure 3. Multi-axial stress tensor when torque and bending moment is applied.

Sampling Methodology

Specimens were selected out of the genus of ash tree, specifically *Fraxinus uhdei*, with branch diameters that ranged from 1.52 cm to 5.2 cm. After selecting the specimens to be tested, researchers cut the branches from healthy trees only (i.e., branches with decay, cracks, or other defects were not sampled). The experiments were performed within 1–2 hours after collection to avoid branch drying, since the properties of strength will change dramatically as considerable time elapses.

For testing, the sample lengths L_2 varied from 0.5 m to 1.5 m. For small diameter branches (1.5–3 cm), a small lever arm of 0.5 m was preferred, so as to avoid substantial deflection of the branch; for branches with medium diameter (3–4 cm), the lever arm selected was 1 m; and for the large diameter branches (4–5.2 cm), the lever arm of 1.5 m was used. The reason for increasing the lever arm was to facilitate the reaching of the branch failure stress value with a reasonable force F_2 applied.

Bending and Torsional Tests

For both bending and torsion tests, each specimen was slowly loaded to the point of failure; load cells were used to monitor the load as a function of time. Then the stress value for both tension due to bending moment and shear due to torsion was measured. Prior to testing, the load cells were calibrated with a known weight of 22.66 kg.

To ensure the calculation of bending moment and torque accuracy, the perpendicularity between the axis of the branch and the applied bending force F_2 was carefully maintained, as well as the axis perpendicularity of the wrench to the force F_1 that generates torque to the axis of the branch, as depicted in Figure 4.

The branch was stripped of bark before the test was performed to prevent the wrench from sliding, and the cross-sectional properties were then measured (polar moment of inertia I_p and the diameter d) in the zone where failure was to be induced, as seen in Figure 4.

The branch was constrained on one side while the load (with magnitude F_2) was applied at a distance (L_2) to induce a bending moment (M) at the failure point, which tried to rotate the branch around the y axis, and another load (F_1) was applied perpendicularly to the sleeve of the wrench at a distance (L_1) to induce torque (T) around the axis of the branch, hence the wrench tending to rotate the branch around the x axis.

A series of 51 samples were taken to the point of failure by a combination of torque and bending moments. Ten tests corresponded to pure bending moment. Ten more tests were performed, applying pure torque without any bending moment. The remaining tests were performed by a combination of loads with bending moment and torque.

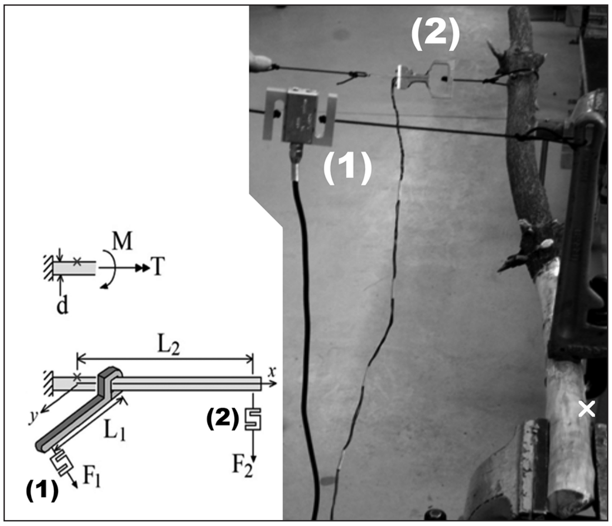

Figure 4. Experimental setup. The cross represents where the failure will be induced; force F_1 induces torque, measured with load cell (1) to the branch, while force F_2, measured with load cell (2), induces a bending moment *M* on the branch with length L_2 and diameter *d*.

Tension Test

A single tension test is presented here to complete the description of the set of loads that can be applied to the branch. This test was performed by pulling a 45 cm long branch with an average diameter of 4.8 cm.

Stresses Calculation

Bending stress σ is calculated as follows:

$$[1] \qquad \sigma = \frac{Mc}{I}$$

where M is the bending moment, c is the distance for the neutral fiber plane to reach the point of failure, which equals half of the branch diameter $d/2$, and I is the moment of inertia, which equals $\pi d^4/64$, assuming the cross section is circular.

Shear stress τ is calculated as follows:

$$[2] \qquad t = \frac{Tr}{I_p}$$

where T is the torsion applied, r corresponds to $d/2$, and I_p is the polar moment of inertia, which is equal to $\pi d^4/32$, also assuming a circular cross-sectional area.

Von Mises-Hencky's Failure Criterion

The first criterion considered in this study is the Von Mises criteria. This failure criterion is applied to isotropic and ductile materials, which is based on the comparison between the second invariant of the stress tensor at limit strength. Although the branches were not an isotropic material, because the wood was not equally distributed along the branch and the fibers possessed different properties and were subject to forces from different orientations, this criteria is advantageous by characterizing the ultimate stress value with only one parameter.

Distortion energy theory (Von Mises 1913) establishes a failure yield criterion expressed as:

$$[3] \quad S = \sqrt{\frac{1}{2}\left[\begin{array}{l}\left(\sigma_{xx}-\sigma_{yy}\right)^2+(\sigma_{xx}-\sigma_{zz})^2+\cdots \\ (\sigma_{yy}-\sigma_{zz})^2+6\left(\tau_{xy}^2+\tau_{xz}^2+\tau_{yz}^2\right)\end{array}\right]}$$

For the stress tensor that considered in this investigation, there was no stress in the z and y direction ($\sigma_{yy} = \sigma_{zz} = 0$), or shear stress t_{xz} and t_{yz}, thus the expression can be simplified as:

$$[4] \quad \left(\frac{\sigma}{S}\right)^2 + 3\left(\frac{\tau}{S}\right)^2 = 1$$

where S is the strength of the material, which has to be obtained experimentally.

The failure criteria for brittle materials, behaving similar to ceramic materials, can be neglected, as it depends on the principal stress values, and the wood species studied here are very flexible and not fragile at all.

Tsai-Hill Failure Criterion

A second criterion, the Tsai-Hill criterion, is used for orthotropic composite materials and was used in the study to include both torsion and bending moment. In 1950, Hill reported a theory of failure for plastic anisotropy, based on Von Mises-Hencky's. Hill's work assumed that the yield stresses were the same in tension and compression. His formulation contained an interaction between stresses that were the same in tension and compression. By containing an interaction between stresses, his formula therefore involved combined failure modes. In 1965, Azzi and Tsai adapted Hill's theory for composites. The Tsai-Hill failure criterion is expressed as:

$$[5] \quad \left(\frac{\sigma_x}{X}\right)^2 - \frac{\sigma_x \cdot \sigma_y}{X \cdot Y} + \left(\frac{\sigma_y}{Y}\right)^2 + \left(\frac{\tau_{xy}}{Z}\right)^2 = 1$$

where σ_x is the stress in the x direction, σ_y is the stress in the y direction, and τ_{xy} is the shear stress. Since there is no stress in the y direction in a branch when bending and torsion is applied, then $\sigma_y = 0$, as illustrated in Figure 3. The expression can then be simplified as:

$$[6] \quad \left(\frac{\sigma}{X}\right)^2 + \left(\frac{\tau}{Z}\right)^2 = 1$$

where X and Z are the ultimate stress values of bending and torsion, respectively, which have to be determined experimentally, and the criteria predicts failure whenever the stress values σ and τ in Equation 6 result in a value higher than one for the left-hand side of the equation.

A branch is a flexible structure that deforms greatly when bending moment M is applied; this bending moment is modeled by applying a perpendicular force, as seen in Figure 5. When large deflection occurs, the effective bending moment caused by the force F in the failure section is given by:

$$[7] \quad M = r \times F$$

where r is the lever arm. When torsion and bending moment are present in the branch, the large deflection due to bending is reduced as torsion makes the structure less flexible; the branch, in consequence, has less capacity to reduce the bending moment as before, and the bending moment caused by the same force F is now:

$$[8] \quad M^* = r^* \times F$$

Note that M^* is greater than M, since r^* is larger when there is little deflection; this is a structural negative effect when torsion is present.

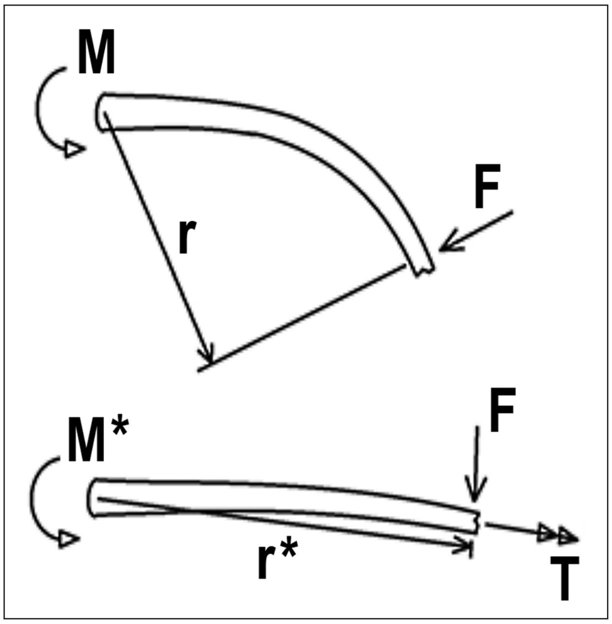

Figure 5. (a) Flexible branch when bending moment is applied, and b) branch becomes stiffer when torque is present and deflects less to bending moment.

RESULTS

For each test sampled, the load applied was recorded as a function of time. As seen in Figure 6, the loads applied (forces that generated torsion and bending moment) were gradually increasing as a function of time until failure was reached, at which point, the loads were applied at a rate of approximately 1.5 kgf per second to ensure inertial loads were insignificant and the dynamic phenomenon was avoided.

In order to obtain a general description of the branch behavior at the failure limiting load, several ratios of bending/torsion loading were applied by digitally monitoring the bending and torsion loads simultaneously using two load cells (data sample interval of 0.005 seconds). This bending/torsion loading ratio is achieved by changing the loading rates of bending and torsion within the interval of 0–1.5 per second, such that different ratios bending/torsion is obtained at the breaking point of each branch sample. The ratio limits correspond to the case of pure bending loading (rate of 1.5 per second) and no torsion loading (rate of zero per second), and the other limit ratio corresponds to pure torsion loading with no bending loading.

The results presented in Figure 7 demonstrate that the Von Mises failure criterion, Equation 4, leads to an incorrect prediction of the experimental data. On the contrary, the results in Figure 8 show the ellipse described by the Tsai-Hill failure criteria (Equation 6), and adjusts to the experimental data for the load combination of shear stress due to torsion and tensile stress due to bending moment.

In the results shown in Figure 8, corresponding to the Tsai-Hill criterion, three ellipses are plotted. The smallest ellipse is the minimum observed load tested that breaks the branch, and should give a conservative failure criterion, where any load combination that falls inside the ellipse does not break the branch. The second ellipse in Figure 8 corresponds to the mean ellipse, using least-squares method, this gives reference to where it is expected that the branch breaks for any combined load. And finally,

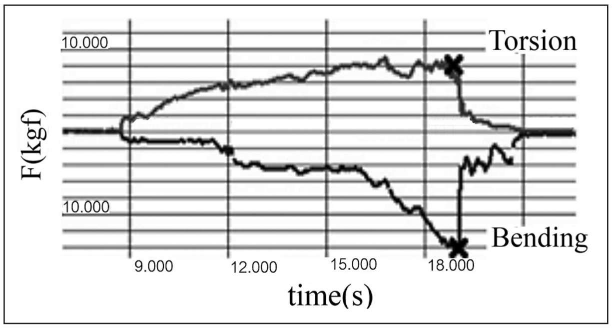

Figure 6. Regular increase of load values applied at an interval rate of 0–1.5 kgf per second; load F_1 is shown as the top solid line (force that induces torsion), and F_2 is shown as the bottom solid line (force that induces bending moment) as function of time for a typical test.

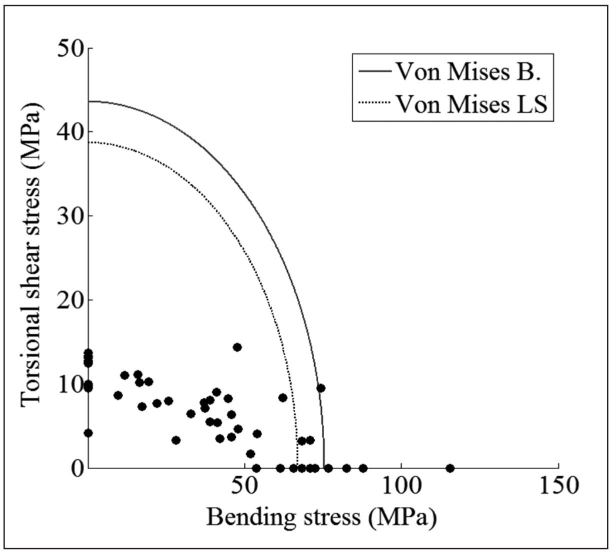

Figure 7. Dotted line: Von Mises criterion using least squares. The solid line represents the Von Mises criterion using only pure bending data for *Fraxinus uhdei*. This plot shows the stress values of torsional stress and bending stress. The *x* axis represents pure bending applied and the *y* axis pure torsion applied.

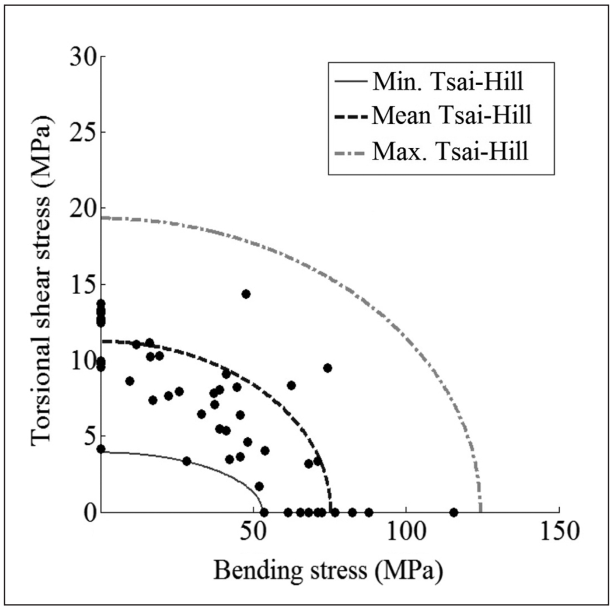

Figure 8. Solid line: Minimum envelope of Tsai failure criterion using X_{min} and Z_{min} values. Dotted line: Tsai-Hill ellipse using least squares. Dashed line: Tsai-Hill ellipse adding to $\bar{X}$ and $\bar{Z}$ values three times the standard deviation of pure bending and pure torsion data for *Fraxinus uhdei*. This plot shows the stress values of torsional stress and bending stress. The *x* axis represents pure bending applied and the *y* axis pure torsion applied.

the third one is an ellipse three times the standard deviation of the pure bending data to the mean X value and similarly for Z value; this gives the ellipse where the branch breaks for any combined load that falls outside such an ellipse with a certainty of 99.5% probability (assuming normal distribution).

For the tree species selected, it was experimentally determined that the X_{min} (minimum value of the maximum ultimate stress values under pure moment) and Z_{min} (minimum value of the maximum stress under pure torsion) values are:

$$X_{min} = 53.6 \text{ MPa}$$
$$Z_{min} = 4.1 \text{ MPa}$$

The selection of X_{min} and Z_{min} are determined such that the experimental data always falls outside the ellipse drawn. A simple rule would be to perform a series of tests applying bending moment only, where the minimum stress value determines the X_{min} value, then perform a series of tests applying torsion only, and the minimum value determines Z_{min}. Alternatively, instead of using the minimum value, a probability density function can be constructed, and the value corresponding to the 1% percentile may be used to obtain the X_{min} and Z_{min} values for the Tsai-Hill criterion.

To estimate the X mean value, denoted by $\overline{X}$, it is calculated that the mean value of the test results be obtained under pure bending only. Experimentally, it is obtained as $\overline{X}$ = 75.5 MPa, similarly for pure torsion, obtained as $\overline{Z}$ = 11.2 MPa. Table 1 summarizes the statistics for the tests under pure bending and for the pure torsion case.

A tension test was performed to complete the set of loads applied to the branch (see Figure 9). The test showed a peak stress value at approximately 24.15 MPa, which corresponded to Point 1 of the Figure 9. In this event, during the test, a cracking sound occurred, indicating that not all fibers have the same flexibility, and that the more rigid fibers were carrying a higher load than the more elastic fibers. After the peak, some rigid fibers were broken and the more elastic fibers started carrying higher load. This implies that the effective area is less than the nominal area. The final stress value of 46.2 MPa, as shown in Figure 9 as Point 2, is lower than the pure bending stress value, with mean stress of 75.5 MPa. Thus, it can be concluded that the tension stress calculated with nominal area does not provide the true stress along the cross section.

Table 1. Maximum, minimum, mean, and standard deviation of the stress values.

	Min psi (MPa)	Max psi (MPa)	Mean psi (MPa)	Std. psi (MPa)
Pure moment	7774 (53.6)	16739 (115.4)	10942 (75.5)	2365 (16.3)
Pure torsion	600 (4.1)	1985 (13.7)	1621 (11.2)	385 (2.7)

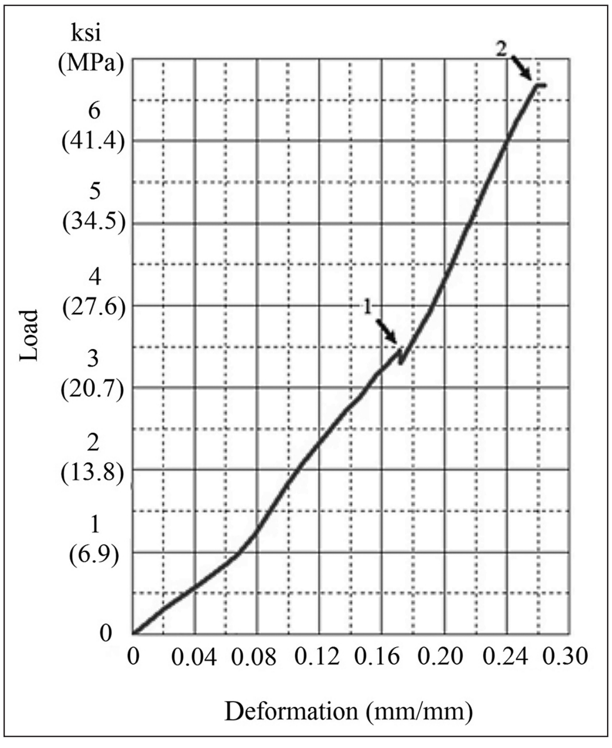

Figure 9. For the tension tests, there were three changes in the slope of load versus deformation as fibers breaks, starting from the stiffer ones to the flexible ones. Point 1 corresponds to failure of the most rigid fibers and Point 2 corresponds to the final stress upon failure of the whole branch.

DISCUSSION

In the presence of torque loads, a considerably lower bending moment was required to break the branch. Torsion affects branch resistance against applied loads in two main aspects. First, torsion displaces the fibers relative to each other, requiring lower stress values than are needed to break the fibers aligned across the branch axis. This is confirmed when calculating the strength due to shear (Z value), which is considerably lower than the strength needed to cut the fibers (X value). Second, torsion reduces branch flexibility, which results in lower deflection and higher bending moment as compared to a very flexible branch with both torsion and bending applied with the same transverse force, similar to the effect of wind across the branch.

The mean ultimate stress values obtained for pure bending $\overline{X}$ = 75.5 MPa, and the mean stress values for pure torsion $\overline{Z}$ = 11.2 MPa, where the ratio of the mean value of pure bending $\overline{X}$ over the mean value for pure torsion $\overline{Z}$ gives a value of 6.74, consistent with calculated values reported out of 178 species (see Niklas and Spatz 2010). Note that the ratio using the Von Mises criterion is fixed to the value of 1.73 (square root of three), a value much lower than what was observed experimentally.

The minimum stress values X_{min} = 53.6 MPa and Z_{min} = 4.1 MPa represent the minimum maximum stress value found on the branch of the *Fraxinus uhdei* tree and should thus be considered as a reference limit for risk assessment of the branch. In a more rigorous approach, the maximum value that the branch can withstand is probabilistic, represented by a probability density function obtained from the recollected experimental data.

As this study focused on branches, the question about the effect of torsion on the trunk arises. It is likely that torque is present when there is interaction between branches and trunk; however, a favorable condition for the trunk is that certain symmetry around the trunk axis is typically present, and this condition tends to cancel the torque of one adjacent branch, when wind blows, with another, opposite adjacent branch. However, the lack of certain symmetry could potentially generate high values of torque in the trunk, and then the Tsai-Hill criteria considering both bending and torsion is appropriate.

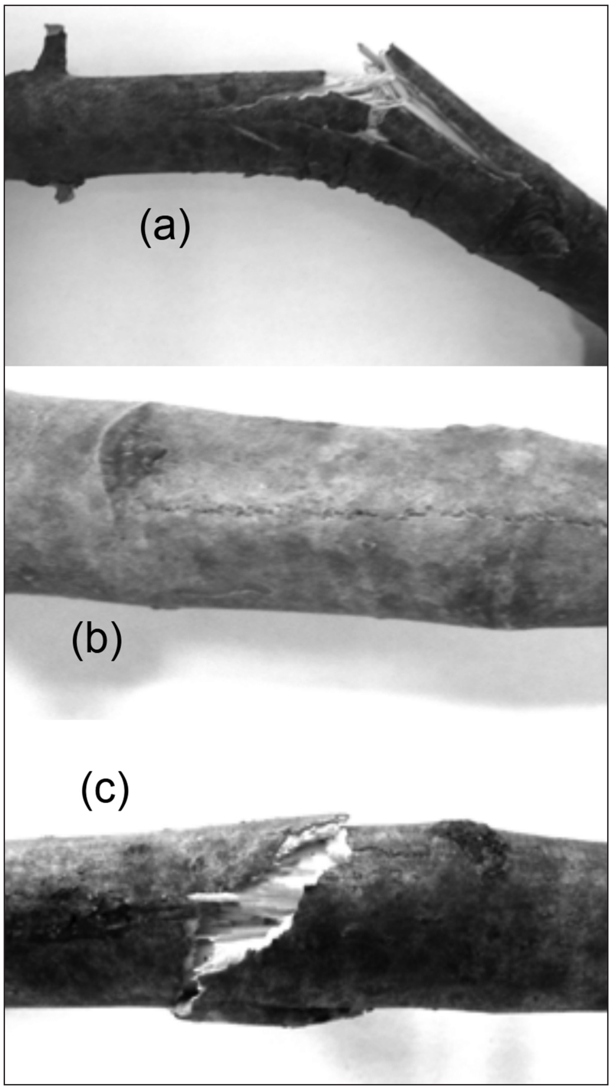

Figure 10. Identification of tree branches failures due to: a) pure bending moment, whereupon tensile stress breaks top fibers and compressive stress crushes the fibers below the neutral fiber, b) pure torsion with a large crack separating the fibers, and c) combination of moment and torsion.

Visual characteristics of the branch failures reveal the type of external forces that were applied leading to failure. Researchers identified the pure bending moment, pure torsion, and the most common combination of loads as illustrated in Figure 10. Thus, it is possible to investigate the type of load that breaks the branch (i.e., failure when pure bending is present as shown in Figure 10a), when pure torsion is applied as shown in Figure 10b, or combination of bending and torsion that caused a branch to break after the event of failure by visualization on the broken branch, as seen in Figure 10c.

To identify when torque loads are potentially present in a branch, adjacent branches to the branch of interest have to be observed carefully. Torque will depend on the perpendicular force to the adjacent branch; for example, it can be identified if two branches shown in Figure 11a, attached to the branch labeled as Branch A, will induce torsion at Section 2 and Section 3. The risk analysis should consider both

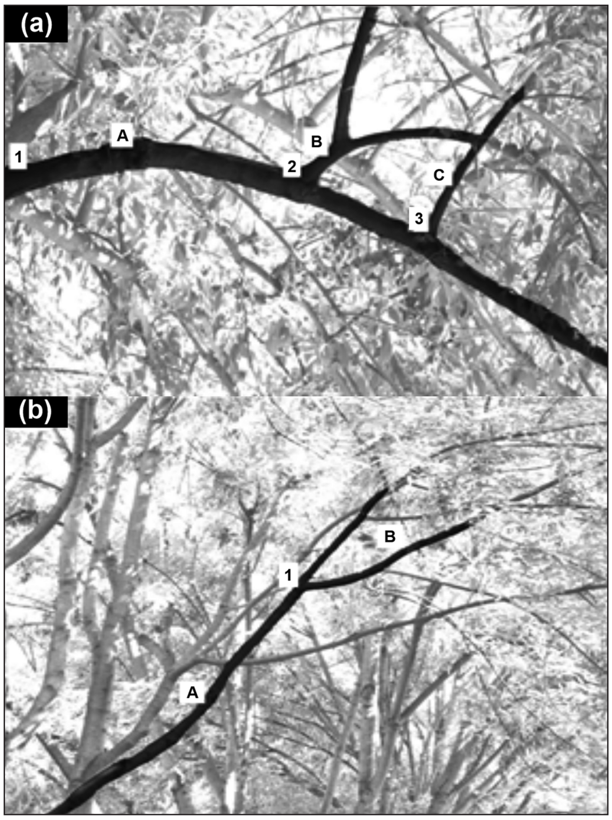

Figure 11. Perpendicular branches to a main branch have the potential to generate high torque loads: a) Branch A is subjected to torque at Section 1 because of the interaction at Nodes 2 and 3 with Branches B and C, respectively, and b) Branch B is at an angle from the main Branch A so as to generate torque when wind forces are present.

torsion and bending moment load to predict failure at Section 1. Another case is shown in Figure 11b, where one adjacent branch is identified and labeled as Branch B, which will generate torsion to Branch A at the attachment Node 1. For this case, bending moment load is dominating; however, torsion load should be considered as well in the failure analysis prediction to give an accurate risk assessment.

CONCLUSION

The effectiveness of the Tsai-Hill failure criteria to predict the reduction of the ultimate stress value for branch failure of *Fraxinus uhdei*. The results clearly show that neglecting the presence of torsion, corresponding to the frame used in the Von Mises criteria, underestimates the risk of branch failure. This analysis has not previously been addressed on healthy branches and for the *Fraxinus uhdei* tree.

The results shown in this study can be complemented with literature providing maximum stress value due to bending and the shear stress due to torsion (see Niklas and Spatz 2010). Such values, and the Tsai-Hill failure criteria, should be used to predict the failure condition for any combination of bending moment and torsion. Although the branches are expected to be largely dominated by the bending moment loads, torque loads should not be ignored in the risk assessment.

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Résumé. L'évaluation d'un arbre repose sur l'identification et l'évaluation des conditions qui permettent de prédire la rupture structurelle. Un arbre est une structure complexe fait de matériaux composites vivants, où l'intéraction des branches due aux forces externes et internes crée des tensions multi- axiales. En considérant le bois comme un matériau orthotrope, les conditions de rupture sont dépendantes des moments de flexion et de torsion auxquels l'arbre est soumis. Il a été démontré que la rupture naturelle de branches de frêne (*Fraxinus* spp.) se produit quand elles sont soumises à la fois à des moments de torsion et de flexion. Les valeurs des contraintes maximales mesurées dans les expériences sont en concordance avec les observations en milieu naturel. Une courbe représentant la dépendance de cette contrainte maximale en fonction des moments de torsion purs, de flexion purs et de la combinaison des deux a été obtenue. Pour les espèces d'arbres testées, les résultats expérimentaux ont montré des valeurs maximales de contraintes, cohérents avec les rapports précédents, de x = 75,5 MPa pour la flexion pure et z = 11,2 MPa pour la torsion pure. La méthodologie présentée dans cette étude peut être appliqué à toutes les espèces d'arbres.

Zusammenfassung. Baumbegutachtung hängt ab von der Identifizierung und Beurteilung struktureller Konditionen vor Vorhersage von Baumversagen. Der Baum ist eine komplexe Struktur aus lebendigen Baustoffen, wo multi-axiale Stressbedingungen in der Interaktion der Äste zwischen externen und internen Kräften auftreten. Versagenskriterien beinhalten Biegemoment und Torsion, während das Holz ebenso als orthotrophisches Material betrachtet wird. Äste von Eschen (*Fraxinus* spp.), die unter natürlichen Bedingungen brachen, zeigten hierbei die gleichzeitige Einwirkung von Torsion und Biegung während des Versagens. Maximale Stresswerte, die in kontrollierten Versagensexperimenten gemessen wurden, zeigten genaue Übereinstimmung mit den verwendeten Kriterien in der Studie, so dass eine Kurve entstand vom absoluten Biegemoment-Versagen bis zum reinen Torsionsversagen und Kombinationen aus beiden. Aus diesen Experimenten wurden ultimative Werte für die getesteten Baumarten bestimmt: $\overline{X}$ = 75.5 MPa für reine Biegung und $\overline{Z}$ = 11.2 MPa für reine Torsion, was völlig übereinstimmt mit den vorangegangenen Berichten. Die hier in dieser Studie verwendete Methode kann auf jede Baumart angewendet werden.

Resumen. La evaluación de los árboles para predecir su falla se basa en la identificación y evaluación de las condiciones estructurales. El árbol es una estructura compleja de compuestos vivos donde existen condiciones de estrés multi-axial en la interacción de las ramas entre fuerzas externas e internas. Los criterios de falla incluyeron momento de flexión y torsión, teniendo en cuenta también la madera como material ortotrópico. Las ramas de fresno (*Fraxinus* spp.) que fallaron en condiciones naturales demostraron estar sometidas a la vez a la torsión y a la flexión. Los valores máximos de tensión medidos en los experimentos controlados de falla demostraron excelente acuerdo con los criterios utilizados en el estudio, obteniendo de este modo una curva que se extiende desde el momento de flexión, pasando por la torsión, y la combinación de ambos. A partir de estos experimentos, los valores determinados para los árboles probados fueron $\overline{X}$ = 75.5 MPa para pura flexión y $\overline{Z}$ = 11.2 MPa para pura torsión, lo que concuerda con reportes anteriores. La metodología mostrada en este estudio se puede aplicar a cualquier especie de árbol.